This Week's Citation Classic

Howard L N. Note on a paper of John W. Miles. *J. Fluid Mech.* **10**:509-12, 1961.
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The theorem that local Richardson number > ¼ implies stability of a stratified shear flow, established by Miles in a preceding paper,[1] is here given a simpler and more general proof. Also, the complex wave velocity of any unstable mode in such a flow must lie in a certain semicircle. [The *SCI*® indicates that this paper has been cited in over 155 publications since 1961.]

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"In 1920, L.F. Richardson published a paper[2] in which, on the basis of a combination of physical and mathematical considerations, he argued that turbulence in the atmosphere should die away if the *static stability*, measured by $-g\varrho'/\varrho$ ($\varrho(z)$ giving the dependence of mean density on height) would exceed the square of the mean shear. Richardson's argument was not really a mathematical demonstration, but it seemed physically quite convincing and was an important advance in the understanding of atmospheric turbulence—for the first time the importance of the ratio of static stability to shear squared (now called the *Richardson number*) was clearly recognized.

"In 1931, G.I. Taylor[3] finally published some studies (mostly done in 1914!) of the *stability* of certain idealized stratified flows. From these examples, in which he was able to solve the relevant equations exactly, he conjectured that in general such flows would be stable if $-g\varrho'/\varrho$ would exceed $\frac{1}{4}U'^2$—as we would now say, if the local Richardson number exceeded ¼. Stability of a parallel flow is not exactly the same thing as the decay of turbulence, but Taylor's investigation seemed to fit in pretty well with Richardson's ideas.

"Some time in 1960, J.W. Miles found a proof of Taylor's conjecture, valid for a large class of stratified flows, and wrote a paper about it.[1] I saw this paper before publication, I think because Miles sent me a preprint. I was impressed by the analytical ingenuity of Miles's argument, but also got the feeling that the generality and simplicity of the result hinted that a less analytically detailed approach might be found. At this time, I had been actively working on hydrodynamic stability problems for several years, having been introduced to this subject by C.C. Lin when I joined the Massachusetts Institute of Technology faculty in 1955. I tried to find out if any integral relations, such as the one used to show Rayleigh's inflection point theorem for homogeneous parallel flow, could shed some light on the stratified case. After a certain transformation of the stability equation (probably motivated largely by the structure of Rayleigh's argument—I have never found a good physical motivation), I was delighted to find such an integral relation which led almost immediately to exactly Miles's result. In the course of this I also found a similar relation which could be used to establish a fact about stratified shear flow, the semicircle theorem, that rather elegantly extended and completed a previous result of Synge[4] for homogeneous flow.

"I wrote to Miles about these matters, and he then suggested, to me and to the editor of the *Journal of Fluid Mechanics*, Batchelor, that I write up a note to be published with his paper which by then had been accepted. Batchelor went along with this suggestion, and the two papers appeared together. Both the Richardson criterion and the semicircle theorem later turned out to have analogs in other stability problems; this perhaps has something to do with the number of citations this paper has received.

"A recent monograph covering work in this field is *Hydrodynamic Stability*."[5]

1. **Miles J W.** On the stability of heterogeneous shear flows. *J. Fluid Mech.* **10**:496-508, 1961.
   [The *SCI* indicates that this paper has been cited in over 175 publications since 1961.]
2. **Richardson L F.** The supply of energy from and to atmospheric eddies.
   Proc. Roy. Soc. London Ser. A **97**:354-73, 1920.
3. **Taylor G I.** Effect of variation in density on the stability of superposed streams of fluid.
   Proc. Roy. Soc. London Ser. A **132**:499-523, 1931.
4. **Synge J L.** The stability of heterogeneous liquid. *Trans. Roy. Soc. Can.* **27**:1-18, 1933.
5. **Drazin P G & Reid W H.** *Hydrodynamic stability.* Cambridge, England: Cambridge University Press, 1981. 525 p.