Adams J F. On the non-existence of elements of Hopf invariant one.
Ann. Math. 72:20-104, 1960.
[Trinity Hall, Univ. Cambridge, Cambridge, England]

The problem of the existence of elements of Hopf invariant one is finally settled (in the negative). Although such elements do not exist, their nonexistence is related to interesting topological phenomena which do exist, such as nonzero differentials in the 'Adams spectral sequence.' The proof requires the construction and use of a theory of secondary cohomology operations, and much homological algebra. [The *SCI*® indicates that this paper has been cited in over 175 publications since 1961.]

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"This work was done at the Institute for Advanced Study in Princeton, New Jersey, in the year 1957-1958. New members of the institute were welcomed in a body by J.R. Oppenheimer; he gave them an invitation, which sounded very good, to cocktails in his house any evening—provided, of course, that it would not be convenient if we all came during the first week. Before the first week was out, the young mathematicians had enough social life of their own to disregard the director. Smullyan was a conjuror of a professional standard; and I have photographs of Serre and Whitney performing feats of skill suited to mountaineers at parties far from any mountains.

"I would like non-topologists to think of 'an element of Hopf invariant one' as a curiosity of nature, comparable with the duckbill platypus. Since the 1930s, we knew that such curiosities did exist in dimensions n=1,2,4, and 8. It was most provoking not to know whether or not there were any more. Even in mathematics there are prizes for guessing the right answer (but you have to be able to prove it). It is about par to guess by generalising from three special cases you understand. When I guessed that there were no more of these curiosities, my guess was pretty insecure, for it rested on the evidence of only one special case (Toda[1] had done the case n=16).

"I suppose that my paper is cited partly on grounds of fact, and partly on grounds of method. When mathematicians, in the course of a strict proof, need some known fact they don't stop to prove, they are supposed to cite a reference for it. I should think this accounts for a lot of references to my paper; it proves some facts which come in handy in other proofs. On the other hand, if you solve an old problem you probably do it by introducing new methods or ideas which other workers with other problems may find useful or enlightening. So I suppose that some of the references to my paper come from people who found it handy as a source, not only for methods of proof, but perhaps even for ways of understanding the difficulties they were trying to tackle.

"About honours, IBM has prepared a chart of the history of mathematics from the year 1000 to the 1900s. If you look at the 1978 edition, the year 1960 has a box about six inches down in which my name appears (with eight others) in type of the smallest size. It seems to be there because of this paper. At the time, I was happy enough to get a junior job in the University of Cambridge. A general survey of the subject can be found in *Algebraic Topology—A Student's Guide*."[2]

1. **Toda H.** Le produit de Whitehead et l'invariant de Hopf. *C. R. Acad. Sci.* 241:849-50, 1955.
2. **Adams J F.** *Algebraic topology—a student's guide*. Cambridge, England: Cambridge University Press, 1972. 300 p.