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This Week's Citation Classic

Parthasarathy K R. *Probability measures on metric spaces.*
New York: Academic Press, 1967. 276 p.
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The structure of Borel sets in complete and separable metric spaces, convolution of probability measures in groups and Hilbert spaces, and generalisations of classical limit theorems for sums of independent random variables in such spaces are investigated. [The *Science Citation Index*® (*SCI*®) and the *Social Sciences Citation Index*® (*SSCI*®) indicate that this book has been cited in over 450 publications since 1967.]

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"In 1956, V.S. Varadarajan and I joined the Indian Statistical Institute at Calcutta and came into contact with R. Ranga Rao. A few years later another young probabilist, S.R.S. Varadhan, joined our group. Calcutta was a difficult city and the bamboo sheds in which we lived did not make life easier. However, the institute had a good library and international contacts. We forgot all the misery of life by plunging into mathematical research.[1,2] Soon we became familiar with the entire spectrum of work done by the Russian school of probabilists under the leadership of A.N. Kolmogorov. Varadarajan had already rediscovered many results of Yu.V. Prohorov on the interplay between topology and measure and taught us about this topic. Later he left for Princeton in order to pursue his research in a different direction and the remaining three of us embarked on a project under the leadership of Ranga Rao to generalise systematically the contents of the famous book on limit theorems by Kolmogorov and B.V. Gnedenko[3] to 'sums' of independent random variables with values in a group or Hilbert space.[1,2] This led to new notions like shift compactness, Gaussian distribution on a group, and finally, Lévy-Khinchine representations in the new setting. The Lévy-Khinchine representation in a Hilbert space is also called the Varadhan representation.[4]

"I spent the year 1962-1963 in Moscow and found that many of our results were being rediscovered. Hence, I gave a few seminars and determined in my mind to put down these results in book form. In collaboration with V.V. Sazonov, I made some minor improvements in the theory developed at Calcutta.[5] In November 1963, I returned to Calcutta but, owing to the influence of the US on Varadarajan, our research work took an entirely different direction. Owing to marriage and the difficulties of daily life in Calcutta, all the members of this quartet emigrated to the West. I landed in Sheffield in 1965 and found that the atmosphere under the leadership of J. Gani was very pleasant and at the same time competitive. To make my mark I organised a seminar on the Calcutta results and summarised them in my notes. The American probabilist E. Lukacs visited the department and in the course of a chat proposed that the notes be published in book form. With very little change the book was published in 1967. Soon after that, I was rewarded with a personal chair at the University of Manchester.

"This book has been cited for several reasons. The first chapter of the book which was written only in order to include some technical points assumed, rather strangely, a special importance in mathematical economics and other social sciences. Probably for the first time a summary of the well-known work of the Polish school on Borel and analytic sets appeared here in an easily comprehensible form and made the book popular among nonprobabilists. For a more recent review see *Probability Measures on Locally Compact Groups.*"[6]

1. **Parthasarathy K R, Ranga Rao R & Varadhan S R S.** On the category of indecomposable distributions on topological groups. *Trans. Amer. Math. Soc.* **102**:200-17, 1962.
2. ———. **Probability distributions on locally compact Abelian groups.** *Ill. J. Math.* 7:337-69, 1963.
3. **Gnedenko B V & Kolmogorov A N.** *Limit distributions for sums of independent random variables.* Reading, England: Addison-Wesley, 1954. 264 p.
4. **Varadhan S R S.** Limit theorems for sums of independent random variables with values in a Hilbert space. *Sankhyā* **24**:213-38, 1962.
5. **Parthasarathy K R & Sazonov V V.** On the representation of infinitely divisible distributions on locally compact Abelian groups. *Theor. Probab. Appl.—Engl. Tr.* **9**:118-22, 1964.
6. **Heyer H.** *Probability measures on locally compact groups.* Berlin: Springer-Verlag, 1977. 531 p.