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# This Week's Citation Classic

**Helstrom C W. Image restoration by the method of least squares.**
*J. Opt. Soc. Amer.* 57:297-303, 1967.
[Westinghouse Research Laboratories, Pittsburgh, PA]

**The restoration of optical images blurred by diffraction or motion, or the unfolding of data that have been convolved with an instrumental window function, is viewed as the minimum-mean-square-error estimation of a signal that has been filtered and corrupted by random noise. [The *SCI*® indicates that this paper has been cited in over 125 publications since 1967.]**

Carl W. Helstrom
Department of Electrical Engineering
& Computer Sciences
University of California
La Jolla, CA 92093

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"While working in the mathematics department at Westinghouse Research, I was occasionally asked how best to 'deconvolve' an instrumental window function from data observed in nuclear, atomic, or optical spectroscopy. Straightforward solution of the pertinent integral equation yields a spectrum in which details are often masked by erratic fluctuations. Having worked on problems in signal detection and estimation, I realized that here was an analogue of estimating a signal that had passed through a linear filter and been corrupted at the output by noise. The errors in the data, the counterpart of white noise, were excessively amplified by the deconvolution method. Instead of trying to solve the integral equation exactly, one should accept that linear estimate of the spectrum that minimizes the mean-square deviation from the true one, and Norbert Wiener had shown how to find it.[1] I wrote a report on this approach early in 1965,[2] but something so elementary could hardly be published except in the context of a neat practical application.

"The only real data to which I applied the method came from an experiment to measure the cross section of the reaction $e + H_2 \rightarrow H^- + H$ as a function of the energy of the electron. The distribution of energies of the $H^-$ ions, observed in a mass spectrometer, was the convolution of that cross section with the energy distribution in the electron beam, which corresponded to a window function; was nearly as broad as the peaked cross section function being sought; and was quite inaccurate, having itself been determined experimentally. The physicists did not trust the method enough to report the result of my deconvolution other than qualitatively.[3] This was not an application that lent itself to a clear-cut exposition of the statistical approach.

"For a few years my own research had dealt with optical signal detection, and while scanning the optics literature for related work, I discovered a paper by James Harris on image restoration,[4] which furnished just the application I was looking for. I quickly translated the estimation equations into the two-dimensional domain of optical imagery, wrote it all up, and sent it to the *Journal of the Optical Society of America*. A few weeks later I left Westinghouse for the University of California, San Diego, and there I found Harris and his colleagues assembling an elaborate computer system to sample blurred images and try out a variety of restoration algorithms: further research was clearly in capable hands. In the literature on image processing, which has vastly proliferated since then,[5] my paper may have been cited so often because it got into print early and introduced the use of statistical estimation in simple terms."

1. **Wiener N.** *The extrapolation, interpolation and smoothing of stationary time series.* New York: Wiley, 1949. 163 p.
2. **Helstrom C W.** *The solution of convolutional integral equations involving experimental data.* Pittsburgh, PA: Westinghouse Research Laboratories, 13 January 1965. Research Report 65-1C3-MCONS-R1.
3. **Schulz G J & Asundi R K.** Isotope effect in the dissociative attachment in $H_2$ at low energy. *Phys. Rev.* **158**:25-9, 1967.
4. **Harris J L.** Image evaluation and restoration. *J. Opt. Soc. Amer.* **56**:569-74, 1966.
5. **Huang T S, ed.** *Picture processing and digital filtering.* Berlin: Springer-Verlag, 1979. 297 p.