Adler S L. Axial-vector vertex in spinor electrodynamics. *Phys. Rev.* **177:2426-38, 1969.**
[Institute for Advanced Study, Princeton, NJ]

This paper shows that the axial-vector vertex in spinor quantum electrodynamics has anomalous properties, arising from the presence of closed-loop 'triangle diagrams.' The operator structure of the anomaly in the divergence of the axial-vector current is worked out, and its consequences, in particular for neutral pion decay, are studied. [The *SCI*® indicates that this paper has been cited in over 545 publications since 1969.]

Stephen L. Adler
Institute for Advanced Study
Princeton, NJ 08540

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"This paper originated during a visit to the Cavendish Laboratory in Cambridge, England, during April-May 1968. I had set myself the goal of proving that the axial-vector vertex in spinor quantum electrodynamics is finite when the usual external line wave function renormalizations are included. I had produced a formal inductive proof of finiteness using the naive axial-vector Ward identity and, to be careful, decided to check the lowest nontrivial case, involving triangle graphs, using an explicit calculation of this diagram which had been published by L. Rosenberg.[1] To my surprise, I found that my formal argument for finiteness failed for these diagrams, as did the naive axial-vector Ward identity. On reexamining the derivation of the naive identity, I found that it used a shift of integration variables inside a linearly divergent integral, an operation which leads to the appearance of an added finite contribution which was omitted in the naive form. I can still recall my astonishment, and that of Sam Treiman (with whom I was sharing an office) when we discussed these results.

"I continued work on the axial-vector graphs during a summer visit to the Aspen Center for Physics. I found that the added term in the Ward identity has a simple operator form, and worked out some of its mathematical properties, including showing that my original conjecture regarding finiteness of the axial-vector vertex is false. In the abstract of my write-up, I referred to the 'anomalous properties' of the axial-vector vertex, and the term 'axial-vector anomaly' has become standard usage. No sooner had I finished my draft when Sidney Coleman arrived in Aspen from Europe, and told me that J.S. Bell and R. Jackiw had simultaneously discovered the anomalous behavior of the triangle graph in an investigation of Sutherland's current algebra theorem on neutral pion decay.[2] I disagreed with their argument that the anomaly could be eliminated by regulators, and wrote an appendix to my manuscript, showing that the anomaly could not be eliminated without spoiling gauge-invariance, unitarity, or renormalizability. Also in the appendix, I used the anomaly and current algebra to derive a formula for the neutral pion decay rate into two photons.

"A remarkable property of the anomaly, which took several years to gain acceptance, is that it is unrenormalized by higher order perturbation theory corrections.[3] As a result, the neutral pion decay formula gives information about the number of quarks participating in the strong interactions, and provides one of the first pieces of evidence for the existence of the color quantum number. Axial-vector anomalies play an important role in the analysis of non-Abelian gauge theories,[4] and there are similar anomalies in the trace of the energy-momentum tensor. These are the main reasons why my paper has been frequently cited."

1. **Rosenberg L.** Electromagnetic interactions of neutrinos. *Phys. Rev.* **129**:2786-8, 1963.
2. **Bell J S & Jackiw R.** A PCAC puzzle: $\pi^0 \to \gamma\gamma$ in the σ model. *Nuovo Cimento A* **60**:47-61, 1969.
3. **Adler S L & Bardeen W A.** Absence of higher-order corrections in the anomalous axial-vector divergence equation. *Phys. Rev.* **182**:1517-36, 1969.
4. **Itzykson C & Zuber J B.** *Introduction to quantum field theory.* New York: McGraw-Hill, 1980. 705 p. Sections 11-5; 12-4; 12-5.