Harding J P. The use of probability paper for the graphical analysis of polymodal frequency distributions. *J. Mar. Biol. Ass. UK* 28:141-53, 1949.
[British Museum, Natural History, London, England]

A graphical method, with examples, was described for estimating the means and standard deviations of the constituent populations of a polymodal distribution. The method was not new but was simple and elegant for solving a mathematically very complex problem often encountered by biologists. [The *SCI*® indicates that this paper has been cited in over 270 publications since 1961.]

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"How strange that this should be a most-cited paper. It is not in my own field of research and I have myself referred to it only once in the last 30 years.[1] I have thought it useful, but, as it was not original, not important. I wrote it as a biologist for biologists describing a method so obvious to mathematicians that they seem never to have bothered to put it into print. I am a zoologist and until my retirement was posted in the Natural History section of the British Museum. It was there, shortly after World War II, that I was sorting a collection of copepods (a kind of water flea) from Lake Titicaca. More than one new species was involved and I was measuring the lengths of specimens and comparing the means and standard deviations of males and females. This was difficult with populations where the sex of the specimens could not be determined visibly. A histogram of the size distribution looked as if it might be bimodal, and as it is normal for a male copepod to be smaller on average than a female, this was to be expected. By chance as I was pondering in a canteen over the possibility of separating the component statistics, I was introduced to the man sitting next to me as a statistician and he was able to tell me of a simple graphical method using probability paper. I had never heard of this form of graph paper but it was soon shown to me and its properties explained.

"I found the method so ingenious and simple that I looked into its history and found that probability paper had been invented early in this century in a study of the probability of flooding by certain rivers in the US. It had been widely used in studies of probabilities and as a graphical method of estimating means and standard deviations.

"It is obscure who first used it to analyse polymodal distributions, a notoriously difficult mathematical task. It may have been someone in Operational Research during the war for one of the uses to which it was put was to detect the bimodal performance of an antiaircraft battery supplied by shells from two separate firms.

"I was fascinated by the method and having solved the problem of the Titicaca copepods looked for other data. Two men who had been my senior officers in the Fisheries Laboratory in Lowestoft where I had started my career had written a paper[2] attempting to analyse a population of about 1,000 similar fish. They had devised a graphical method using normal curves not converted into straight lines and had detected two modes at 15 cm and 12 cm lengths. They had grounds for suspecting that a third small population of small fish might be concealed but their method did not reveal this. When I applied the probability paper method to their data the population was easily resolved into three: 50 percent with mean length of 14.5 cm, 40 percent with mean of 11.85 cm, and 10 percent of 9.15 cm fish, each subpopulation with an appropriate standard deviation. That was when I decided to publish.

"I can make no claim to have developed the method, merely to have contributed to its dissemination. The frequent citation of the paper must be because it describes for biologists in language they can understand a simple way of achieving an end they may have desired but thought involved impossible mathematics. More recent work has been published."[3]

1. **Harding J P.** Lake Titicaca expedition. XV. Crustacea: Copepoda. *Trans. Linnean Soc. London* 1:219-47, 1955.
2. **Buchanan-Wollaston H J & Hodgson W C.** A new method of treating frequency curves in fishery statistics, with some results. *J. Conseil Perm. Int. Explor. Mer* 4:207-25, 1929.
3. **Brassard J R & Correia M J.** A computer program for fitting multimodal probability density functions. *Comput. Program. Biomed.* 7:1-20, 1977.