

Balas E. An additive algorithm for solving linear programs with zero-one variables.
Oper. Res. 13:517-46, 1965.
[Centre of Mathematical Statistics, Rumanian Academy, Bucharest, Rumania]

The additive algorithm was one of the early versions of the approach known as implicit enumeration or branch and bound. It solves a linear program in n 0-1 variables by systematically enumerating an (often small) subset of the 2^n possible binary n -vectors, while using the logical implications of the 0-1 property to ensure that the whole set is implicitly examined. [The *Science Citation Index*[®] (SCI[®]) and the *Social Sciences Citation Index*[®] (SSCI[®]) indicate that this paper has been cited in over, 220 publications since 1965.]

Egon Balas
Graduate School of Industrial
Administration
Carnegie-Mellon University
Pittsburgh, PA 15213

April 26, 1982

"A great many decision problems can be formulated as linear programs in 0-1 variables. Such binary integer programs, or 0-1 programs, serve as mathematical models for capital budgeting, project selection, pipeline or communications network design, structural design, switching circuit design, information retrieval, fault detection, design of experiments, clustering, facility location, truck dispatching, tanker routing, crew scheduling, machine sequencing, and a host of other decision problems involving logical alternatives.

"My own immediate interest in 0-1 programming was aroused by a forest management problem involving alternative road networks. At the time (1962-1963), there were no specialized 0-1 programming algorithms, and the more general field of integer and mixed integer programming was in its infancy. A few years earlier, Gomory¹ had developed a cutting plane algorithm for integer programming, and Land and Doig² had proposed a first version of what later became known as the branch and bound

method for mixed integer programming. Neither of these procedures was available as a computer code (at least not in Bucharest, where I was at the time), and besides both of them were addressing a more general problem than the one I was interested in solving. It seemed to me that the binary nature of the variables ought to be exploited. So I developed a special technique for 0-1 programming, based on systematically assigning the value 0 or 1 to certain subsets of variables and exploring the implications of these assignments by a sequence of logical tests. It used only additions and comparisons (hence the name: additive algorithm). The simplicity of the procedure and the fact that it was so easy to implement on a computer has probably contributed more than anything else to its success and rapid dissemination.

"For successful it was, indeed. After presenting it to a conference, the Third Scientific Session on Statistics, held in Bucharest in 1963, I submitted a brief note on it in French to the *Comptes Rendus de l'Académie des Sciences* (Paris), through the late Grigore Moisil, who sent it to R. Fortet. As Fortet promptly recommended publication, the note appeared in early 1964³ and within a few months stimulated two research reports by the group at SEMA, Paris, that combined my approach with that of Land and Doig. The full English version of my paper appeared in August 1965, and was immediately followed by the papers of Glover, Fleischmann, Petersen, Geoffrion, Lemke and Spielberg, Brauer, Pierce, and others, improving the method, extending or specializing it, reporting on computational experience, etc. An excellent early exposition of the approach is to be found in Geoffrion.⁴ For a more recent and up-to-date survey including some of the work of the above mentioned authors, see my 1975 paper.⁵

"Soon after the publication of my paper I left Rumania and came to the US with my wife and two daughters. Starting a new life as an immigrant is usually an arduous task. In my case, the paper that made my name well known overnight in the profession smoothed over the bumps and made the transition relatively easy."

1. Gomory R E. Outline of an algorithm for integer solutions to linear programs. *Bull. Amer. Math. Soc.* 64:275-8, 1958.
2. Land A H & Doig A G. An automatic method of solving discrete programming problems. *Econometrica* 28:497-520, 1960.
3. Balas E. Un algorithme additif pour la résolution des programmes linéaires en variables bivalentes. *C. R. Acad. Sci.* 258:3817-20, 1964.
4. Geoffrion A M. Integer programming by implicit enumeration and Balas' method. *SIAM Rev.* 7:178-90, 1967.
5. Balas E. Bivalent programming by implicit enumeration. (Belzer J, Holzman A G & Kent A, eds.) *Encyclopedia of computer science and technology*. New York: Dekker, 1975. Vol. 2. p. 479-94.