

This Week's Citation Classic

Aris R. On shape factors for irregular particles—I. The steady state problem. Diffusion and reaction. *Chem. Eng. Sci.* 6:262-8, 1957. [Mathematical Institute, Edinburgh, Scotland]

The effectiveness of a catalyst particle with diffusion and first-order reaction is governed by a parameter proportional to the 'size' of the particle. If that size is taken to be the ratio of the volume to external surface area, the effectiveness factors for all particle shapes come together asymptotically and are reasonably close together over the whole range. [The *SCI*[®] indicates that this paper has been cited over 145 times since 1961.]

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"The main idea of this paper arose when I was working at the (then) Billingham Division of Imperial Chemical Industries in 1952. I had been seconded to the catalyst section to study methods of catalyst testing and quickly ran into the problem of diffusion limitation. A remarkable mathematical physicist, C.H. Bosanquet, was at Billingham at the time, who, when asked if a particular reaction were limited by diffusion, would enquire as to the conditions and then wander around his almost bare office muttering about orders of magnitude followed by one of the Rhodesian Ridgebacks he brought into work. From time to time he would press the top of his head against the wall and come up with a 'scarcely,' 'somewhat,' or 'severely.' Whilst this was great fun and very impressive (for a later check of some of his judgments showed them to be essentially correct under a strict interpretation of his words), it was a difficult act to follow and it was with some relief that I discovered Thiele's classic paper of 1939.¹

In writing up my findings I naturally included an exposition of Thiele's work and happened to notice that the asymptotic effectiveness factors for plate, cylinder, and sphere could be made the same by taking the characteristic dimension of the catalyst particle as the ratio of its volume to its external surface area.

"The result slept in my notebook for five years and saw the light of day through the kind interest of K.G. Denbigh while I was a lecturer in technical mathematics at Edinburgh. In retrospect I seem to have done less than justice to Wheeler, whose paper² I had read in 1952, but had not reread in 1957. His argument is based on his model of pore structure and gives the characteristic particle size as the ratio of volume to area but multiplied by a tortuosity factor of $\sqrt{2}$ for pores. However, this factor could have been absorbed into the definition of the rate constant to give my result. My arguments were mathematical, but, in this paper, somewhat less rigorous than those used later.³ A rather obvious conjecture that, of all shapes of pellet of equal volume, the sphere would be the least effective was later proved very elegantly by Luss and Amundson.⁴

"The engineer's enduring interest in intelligent fudge factors, which wrap up the results of a difficult calculation in simple form, must account for the paper being so often cited. Thiele's original idea of the effectiveness factor was a classic example of this, and my modification fudged in the further complication of shape. Though the materials lay at hand (in a problem of Bird, Stewart, and Lightfoot's preliminary version of *Transport Phenomena*⁵) in the late-1950s, it was not until 1965 that the ultimate normalization of the Thiele modulus, accounting for the kinetics as well as the shape, was published, independently and almost simultaneously, by Bischoff, Petersen, and myself (mention of this is made on page 41 of reference 3). The most recent citation of the paper (or rather of its result as given in reference 3) is by Yortsos and Tsotsis,⁶ who generalize the result for particles with varying activity."

1. Thiele E.W. Relation between catalytic activity and size of particle. *Ind. Eng. Chem.* 31:916-20, 1939. [Citation Classic. *Current Contents/Engineering, Technology & Applied Sciences* 10(2):10, 8 January 1979.]
2. Wheeler A. Reaction rates and selectivity in catalyst pores. *Advan. Catal.* 3:249-300, 1951.
3. Aris R. *The mathematical theory of diffusion and reaction in permeable catalysts.* Oxford: Clarendon Press, 1975. 2 vols.
4. Amundson N.R. & Luss D. On a conjecture of Aris: proof and remarks. *AIChE J.* 13:739-43, 1967.
5. Bird R.B., Stewart W.E. & Lightfoot E.N. *Transport phenomena.* New York: Wiley, 1960. 780 p. [Citation Classic. *Current Contents/Engineering, Technology & Applied Sciences* 10(38):12, 17 September 1979.]
6. Yortsos Y.C. & Tsotsis T.T. Asymptotic behavior of the effectiveness factor for variable activity catalysts. *Chem. Eng. Sci.* 37:237-43, 1982.