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This Week's Citation Classic

Mandelbrot B. The variation of certain speculative prices.
J. Business 36:394-419, 1963.
[Harvard University, Cambridge, MA and Research Center, International Business Machines Corp.]

Changes of commodity and security prices are fitted excellently by the Lévy-stable probability distributions. Their parameter, α, is the intrinsic measure of price volatility. The model also accounts for the amplitudes of major events in economic history. An unprecedented feature is that price changes have an infinite population variance. [The *Social Sciences Citation Index*® (*SSCI*®) indicates that this paper has been cited over 150 times since 1966.]

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"Early in 1961, while on my way to a seminar, I stepped into the office of my host, a Harvard economist. On his blackboard, I noticed a diagram nearly identical to one I was about to draw. His diagram referred to a topic of which I knew nothing: records of the price of cotton. My host had given up his attempt to model this phenomenon, and he challenged me to take over.

"In a few weeks, I had succeeded by introducing a radically new hypothesis. I preserved the *random walk* hypothesis, that the market is like a lottery or a casino, with prices going up or down as if determined by the throw of dice. I also preserved the *efficient market* hypothesis, that the market's collective wisdom takes account of all available information, hence, the price tomorrow and on any day thereafter will *on the average* equal today's price. The third basis of the usual model is the hypothesis that price changes follow the Gaussian distribution. All these hypotheses, due to Louis Bachelier,[1] were first faced seriously in 1960. The resulting theory, claiming that price (or its logarithm) follows a Brownian motion, would be mathematically convenient, but it badly fails to fit the data. First of all, the records of throws of a die appear unchanged statistically, but in comparison, the records of competitive price changes 'look *non*stationary'; they involve countless configurations that seem too striking to be attributable to mere chance. A related observation: the histograms of price changes are very far from the Galton ogive; they are long-tailed to an astonishing degree, due to large excursions whose size is obviously of the highest interest.

"My model replaces the customary Gaussian hypothesis with a more general one, while allowing the population variance of the price changes to be infinite. The model is time-variant, but it creates endless configurations; it accounts for all the data, including both the seemingly nonstationary features, and the seemingly nonrandom large excursions.

"A visiting professorship of economics at Harvard, 1962-1963, was triggered by a preprint that covered cotton and diverse commodities and securities. Also, my paper was immediately reprinted in Cootner's book along with two discussions of it[2-4] and must have affected my becoming Fellow of the Econometric Society. However, after a few further forays[5] my interest was drawn irresistibly away from economics, and toward the very different task of creating the new fractal geometry of nature.[6,7] Having learned to live with the unprecedented infinite variance syndrome had trained me to identify telltale signs of divergence in the most diverse other contexts, and to account for them suitably.

"By its style, my work on prices remains unique in economics: while all the other models borrow the final formulas of physics, I lean on its basic mental tool (invariance principles) and deduce totally new formulas appropriate to the fact that prices are not subjected to inertia. My work is also unique in its power: the huge bodies of data that it fits involve constant jumps and swings, but I manage to fit everything without postulating that the rules themselves shift and change. Thus, my models are acknowledged as having opened a path toward a new and more realistic economics. Nevertheless, the progress of this new economics is slow, due to inherent mathematical difficulties and to my failure to push its development. All too often (though with notable exceptions[8,9]), a publication devoted to the totally discredited Gaussian model quotes my work to show the author's awareness of his work's limitations, and possibly to assuage his conscience."

1. **Bachelier L.** Théorie de la spéculation. *Ann. Sci. École Norm. Supér.* III-17:21-86, 1900.
2. **Mandelbrot B.** The variation of certain speculative prices. (Cootner P H, ed.) *The random character of stock market prices.* Cambridge, MA: MIT Press, 1964. p. 307-32.
3. **Fama E F.** Mandelbrot and the stable Paretian hypothesis. (Cootner P H, ed.) *The random character of stock market prices.* Cambridge, MA: MIT Press, 1964. p. 297-306.
4. **Cootner P H.** Comments on the variation of certain speculative prices. (Cootner P H, ed.) *The random character of stock market prices.* Cambridge, MA: MIT Press, 1964. p. 333-7.
5. **Mandelbrot B B.** Forecasts of future prices, unbiased markets, and "martingale" models. *J. Business* 39:242-55, 1966.
6. ———. *Fractals: form, chance & dimension.* San Francisco, CA: W.H. Freeman, 1977. 365 p.
7. ———. *The fractal geometry of nature.* San Francisco, CA: W.H. Freeman. In press, 1982.
8. **Samuelson P.** Efficient portfolio selection for Pareto-Lévy investments. *J. Finan. Quant. Anal.* 2:107-22, 1967.
9. ———. Efficient portfolio selection for Pareto-Lévy investments. (Stiglitz J E, ed.) *The collected scientific papers of Paul A. Samuelson.* Cambridge, MA: MIT Press, 1972. Vol. 3. p. 861-76.

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