A method is described for obtaining a smooth curve approximating a given data set, x_i, y_i ($i = 1, \ldots , n$), where the ordinates y_i may have errors of the order dy_i. This utility is of interest to experimenters and laboratories in diverse fields. [The _Science Citation Index®_ (_SCI®_) and the _Social Sciences Citation Index®_ (_SSCI®_) indicate that this paper has been cited over 270 times since 1967.]

Christian Reinsch
Institut für Mathematik der
Technischen Universität München
D-8000 München 2
Federal Republic of Germany

May 3, 1982

"Often, laboratory measurements result in a two-column table: on the left-hand side there is the independent variable (time, voltage, channel number, etc.), and on the right-hand side there is the dependent variable which has been measured. Interpolation is necessary if the experimenter wants to present his or her results in graphic form as a curve in a coordinate system. In this situation, it is natural to compensate for possible errors in the measured quantities by not requiring the curve to pass exactly through the data points, but to stay close to them and to avoid any undue oscillations. One used to do this by hand, but in the days of computer-controlled plotters, a mathematical algorithm is required to replace the intuition of a skillful experimenter.

"This paper describes such a method based on a mathematical principle which admits a mechanical interpretation: a flexible wooden or metal strip is fastened by elastic springs to hooks at the given data points; the resulting shape of this strip provides us with the desired smooth curve.

"A computer program for implementing this algorithm is included in this paper. This routine has found its way into the established program libraries like International Mathematical and Statistical Libraries, Inc., Houston, Texas, and is probably used by many without reference. Others, who have gone to the source, must have made this paper a _Citation Classic_. The original algorithm has been improved slightly in the meantime,[1] and the ALGOL program has been replaced by a FORTRAN version, which is available from me.

"A note of caution is necessary, however. Any general smoothing principle is inferior to the least squares fitting of an _ansatz_ which is based on a solid theoretical background. Smoothing as described above should be used only if such information is not available or as a preconditioning of the data for the more refined methods of smoothing."

---

1. Reinsch C H. Smoothing by spline functions. II. _Numer. Math._ **16**:451-4, 1971.