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This Week's Citation Classic

Ablowitz M J, Kaup D J, Newell A C & Segur H. The inverse scattering transform-Fourier analysis for nonlinear problems. *Stud. Appl. Math*. **53**:249-315, 1974. [Clarkson College of Technology, Potsdam, NY]

A class of nonlinear wave equations solvable by methods of inverse scattering are presented. Techniques to obtain solutions to relevant initial value problems are given. Some of the earlier work in the field is reviewed and fit into this framework. [The *SCI*® indicates that this paper has been cited over 285 times since 1974.]

Mark J. Ablowitz
Department of Mathematics
and Computer Science
Clarkson College of Technology
Potsdam, NY 13676

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"Though my doctoral research was on nonlinear waves, it had no direct relationship to the topic covered in the cited paper; i.e., on 'soliton theory' (solitons are special localized nonlinear waves). However, I still vividly remember reading a remarkable paper[1] by a group of scientists from Princeton. In that paper a certain well-known nonlinear wave equation was solved by a method which previously had been used in a completely different context and field: namely, inverse scattering in quantum mechanics.

"Subsequent to my PhD, I left the Massachusetts Institute of Technology to join the faculty of mathematics at Clarkson College where Alan Newell, also a former student of my adviser, David Benney, had recently gone. The same year Harvey Segur came to Clarkson from the California Institute of Technology, and jointly the three of us began a weekly seminar series on nonlinear waves. Solitons were a topic at our seminar, but open problems were not yet clear to us.

"During that year Newell arranged with the American Mathematical Society to hold a conference on nonlinear wave motion at Clarkson in July 1972. Many of the people who had done early work in this area were to come. In particular, Martin Kruskal came from Princeton and gave a very stimulating series of lectures. Moreover, at this conference it was announced that a Russian group had solved a second physically interesting nonlinear wave equation[2] by making use of the ideas in reference 1, a fact which further prompted our interest. One equation might be a fluke, but not two.

"It turned out that just after the Clarkson conference, both Kruskal and I were invited to Dartmouth College to take part in a summer applied mathematics program. We worked actively together on an aspect of soliton theory. By the time I was ready to return to Potsdam, we had essentially completed our joint project. Upon returning to Clarkson our small seminar series began again in earnest, this time with a fourth faculty member, David Kaup from the physics department, jointly we began working on still another physically significant nonlinear wave equation (the so-called sine-Cordon equation) to apply these new methods to. In fact, Kruskal had lectured on aspects of this equation, and our joint project was related to this equation. After some months of very hard work we cracked this problem.[3] What emerged after this was, in fact, a whole class of nonlinear wave equations solvable by using inverse scattering, and expressible in a rather compact way. It was clear to us that the method had a direct analogy to the well-known method of Fourier transforms for linear problems. Hence we termed the method 'the inverse scattering transform,' a name which is still used frequently in the literature.

"As it turned out the field was ready for this conceptual jump, and our paper served as both a stepping stone to more difficult problems, as well as an indepth review and extension of what had been done until that time."

1. **Gardner C S, Greene J M, Kruskal M D & Miura R M.** Method for solving the Korteweg-deVries equation. *Phys. Rev. Lett*. **19**:1095-7, 1967.
2. **Zakharov V E & Shabat A B**. Exact theory of two-dimensional self-focusing and one-dimensional self-modulation of waves in nonlinear media. *Sov. Phys.-JETP Engl. Trans*. **34**:62-9, 1972.
3. **Ablowitz M J, Kaup D J, Newell A C & Segur H.** Method for solving the sine-Gordon equation. *Phys. Rev. Lett*. **30**:1262-4, 1973.