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This Week's Citation Classic

Harrington R F. Matrix methods for field problems.
Proc. IEEE 55:136-49, 1967.
[Department of Electrical Engineering, Syracuse University, Syracuse, NY]

This paper gives a unified treatment of matrix methods useful for field problems. The basic mathematical concept is the method of moments, by which the functional equations of field theory are reduced to matrix equations. Several examples of engineering interest are given to illustrate the procedure. [The *SCI*® indicates that this paper has been cited over 110 times since 1967.]

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"The above cited paper unifies numerical solution techniques for electromagnetic field problems into a general procedure called the 'method of moments.' This method is basically the projection of a functional equation onto a finite dimensional vector subspace, resulting in matrix equations which can be solved by the algorithms of linear algebra. The paper has been cited frequently because it was the first to appear in the electromagnetics literature using the general concepts of functional analysis. An expanded exposition of the method appears in the monograph *Field Computation by Moment Methods*.[1]

"The basic idea of taking a linear functional equation and representing it by a linear matrix equation is relatively old. Galerkin, a Russian mechanical engineer, developed his method around 1915, before it had a firm mathematical basis. Quantum mechanics, developed in the 1920s, used many of the ideas of linear spaces and their extension to Hilbert spaces. However, before the advent of the high-speed computer, these methods were not popular because of the tedious computation required for their use. They were often thought of as last resort numerical methods, to be used only if everything else failed. In truth, however, they are no more numerical than other so-called analytical methods, at least if used properly. They merely emphasize a different aspect of mathematics—that of linear spaces and orthogonal projections.

"In the mid-1960s, several researchers started solving the electromagnetic field equations by numerical methods. The accuracy obtained from these numerical solutions was impressive, but, being brought up on variational solutions, I thought that even greater accuracy could be obtained by the latter method. Hence, if I used a variational solution for the current in a stationary formula for scattered field, I should get an order of magnitude higher accuracy for it than obtainable from a numerical solution. I tried it for the simple case of scattering from a cylinder, and to my surprise I got exactly the same answer as obtained from the numerical solution.

"During the early-1960s, I taught a course on the use of linear spaces for applied mathematics. It became apparent to me that Galerkin's method was formally equivalent to the Rayleigh-Ritz variational method, and also to Rumsey's reaction concept.[2] But the numerical methods being used by researchers in electromagnetic theory were not really Galerkin's method. They used the apparently cruder methods, such as subsectional expansion and point matching. Were these also variational methods?

"The answer was yes, at least in concept. There was no good reason why one had to choose expansion and testing functions to be the same, as was done in both Galerkin's method and in the Rayleigh-Ritz variational method. It was easier to prove mathematical theorems when they were the same, but it made solutions more difficult to calculate. One was really free to choose expansion and testing functions separately for computational convenience, and still claim that the solution was stationary in form. Next came the question as to what to call the general method. After a search of the literature, I decided that the exposition most closely analogous to what I was using was that given by Kantorovich and Akilov.[3] They called it the 'method of moments,' and hence that is the name I chose."

1. **Harrington R F.** *Field computation by moment methods.* Orlando, FL: Krieger, 1981. 238 p.
2. **Rumsey V H.** The reaction concept in electromagnetic theory. *Phys. Rev.* 94:1483-91, 1954.
3. **Kantorovich L V & Akilov G P.** *Functional analysis in normed spaces.* Oxford: Pergamon, 1964. p. 586-7.