CC/NUMBER 45
NOVEMBER 9, 1981

This Week's Citation Classic

Brandow B H. Linked-cluster expansions for the nuclear many-body problem. *Rev. Mod. Phys.* **39**:771-828, 1967.
[Niels Bohr Inst., Univ. Copenhagen, Denmark and Lab. Nuclear Studies, Cornell Univ., Ithaca, NY]

Linked-cluster expansions are derived for open-shell many-fermion systems, for energies, wave functions, expectation-values, and effective transition operators. The open-shell energy expansion is cast in the form of a 'first principles' effective Hamiltonian. These expansions provide a formal justification for the nuclear shell model. [The *SCI*® indicates that this paper has been cited over 410 times since 1967.]

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July 1, 1981

"This work evolved tangentially out of my doctoral thesis, which was part of a general program (under Hans Bethe at Cornell University) to try to explain low-energy nuclear physics in terms of the Brueckner-Bethe-Coldstone theory of nuclear matter. Early in the game, I became aware that our enthusiasm for this approach was not widely shared among the nuclear physics community. The main reason, it seemed to me, was that the basic mathematical technique, the linked-cluster expansion of Goldstone, was considered quite mysterious. One of my goals was thus purely pedagogical, namely, to try to clarify the physical content of the Goldstone expansion by means of a new and far more elementary derivation. I worked out a purely time-independent method which I was quite proud of, and I still feel that this is the most elementary derivation available.

"About this time, I discovered the very elegant and significant paper of Bloch and Horowitz,[1] which presented a partially-linked perturbation expansion for open-shell nuclei. Their result was cast in the convenient and suggestive form of an effective Hamiltonian acting within a 'model' configuration space of limited dimension. Furthermore, they managed to cleanly separate the energy contribution of correlations within the closed-shell 'core' from the (frequently much smaller) contribution due to the valence particles. The form of their result was, therefore, tantalizingly close to that of the conventional nuclear shell model. I was soon able to reproduce their result by means of my elementary approach.

"As an NSF postdoctoral fellow at the Niels Bohr Institute in Copenhagen, Denmark, my first project was to write up these pedagogical linked-cluster studies. I also tried repeatedly to obtain a fully-linked analog of the Bloch-Horowitz result, which could thereby provide a respectable formal justification for the nuclear shell model. After some six months of sporadic efforts, I hit upon the 'folded diagram' representation for degenerate Rayleigh-Schroedinger perturbation theory, and found that it did indeed lead to complete cancellation of all unlinked diagrams. The result now looked essentially the same as the familiar shell model, except for the occurrence of (hopefully small) effective interaction terms involving more than two valence particles. Further effort led to analogous fully-linked results for expectation values and effective transition matrix elements for arbitrary physical operators, thereby providing a 'complete set' of basic formal tools for understanding bound-state aspects of the nuclear shell model.

"The results have been widely used in theoretical nuclear physics, as described in an international conference proceedings,[2] and in a review article.[3] In recent years, some members of the quantum chemistry community have also begun to apply this formalism. I have therefore prepared several introductory papers for this community, the latest of which[4] also contains the most up-to-date survey of later developments in this field. It is now apparent that this formalism provides a general theoretical foundation for effective Hamiltonians in many-body systems, so a variety of applications in other areas of physics can also be envisaged."

1. Bloch C & Horowitz J. Sur la detérmination des premiers états d'un système de fermions dans le cas dégénéré. *Nucl. Phys.* **8**:91-105, 1958.
2. **Barrett B R,** ed. *Effective interactions and operators in nuclei: proceedings of the Tucson International Conference on Nuclear Physics.* Berlin: Springer-Verlag, 1975. 339 p.
3. Ellis P J & Osnes E. An introductory guide to effective operators in nuclei. *Rev. Mod. Phys.* **49**:777-832, 1977.
4. Brandow B H. Formal theory of effective n-electron Hamiltonians. *Int. J. Quantum Chem.* **15**:207-42, 1979.