

# This Week's Citation Classic

Lodge A S. *Elastic liquids: an introductory vector treatment of finite-strain polymer rheology*. London: Academic Press, 1964. 389 p. [Manchester College of Science and Technology, England]

**Vectors embedded in a homogeneously-deforming viscoelastic continuum are used to give an elementary treatment of strain, stress, and rheological behavior that requires no knowledge of tensor analysis. The essentially novel features of polymer viscoelasticity are rapidly derived from first principles. [The SOI<sup>®</sup> indicates that this book has been cited over 370 times since 1964.]**

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"In 1948, after completing a D. Phil, at Oxford in the theory of elementary particles, I joined the British Rayon Research Association (Manchester) to work in polymer rheology with K. Weissenberg. He described his then controversial, now orthodox, view that finite-strain elastic effects dominate the flow behavior of polymeric *liquids*. Until Manchester accommodations became available, I stayed in Oxford where, by chance, I attended a seminar on 'Proofs of Relativistic Invariance by the Method of Point Transformations' given by an American, Crosby. This suggested to me that the simplest way to describe viscoelastic continua mathematically was by introducing 'body tensor fields' defined over the deforming body regarded as a geometric manifold in its own right.<sup>1</sup> Simultaneously and independently, Oldroyd developed a closely-related 'convected component' formalism in a paper<sup>2</sup> of wide-ranging, fundamental importance. The body tensor formalism can be regarded as a coordinate-free alternative to the convected component formalism, and gives simpler concepts and proofs than those given by the conventional space tensor formalism used years later by others to 'rediscover' some of Oldroyd's basic results.

"The body tensor formalism, with its use of time-dependent body metric tensor fields, is the

obvious tool to use for theoretical physicists accustomed to the density-dependent space-time metric tensor fields of general relativity, but is still a stumbling block to some engineers and continuum mechanicians, who assert (incorrectly and with no attempt at a proof) that body tensors are space tensors.<sup>3</sup> My body tensor book<sup>4</sup> has yet to attain the popularity of *Elastic Liquids*, and I can count on the fingers of one hand those known to me who would be seen using a body tensor in public.

"In 1960, I moved to the University of Manchester Institute of Science and Technology, department of mathematics, whose chairman, C.J. Kynch, asked me to give evening lectures to a varied industrial audience. In wondering how to communicate quickly the essentially novel aspects of finite-strain polymer rheology to a tensor-free audience, it struck me that, where the basic physics was concerned, nothing would be lost by restricting attention to *homogeneous* flows, for these reflect the varied and striking rheological phenomena exhibited by polymeric liquids. One could retain enough of the body tensor spirit and simplify the mathematics considerably by using Cartesian base vectors *embedded in the deforming continuum*. Elementary vector analysis sufficed to lead one rapidly into the detailed description of finite-strain effects; the more complicated aspects of general tensor analysis and curvilinear coordinate systems were not needed; the subject was thereby opened up to a wider audience. These lectures led to the writing of *Elastic Liquids*.

"*Elastic Liquids* is often cited, I believe, because it unclutters the subject by avoiding all reference to the unwanted and unnecessary rotations of material elements relative to space-fixed directions that are involved in conventional formulations of continuum mechanics. In addition, photographs and experimental data are included for actual polymeric liquids, and simple calculations of basic rheological properties are developed from first principles. In particular, calculations of large elastic recoveries are included with the introduction of new terms: *constrained* and *free recoveries*. *Elastic Liquids*, reviewed by R.B. Bird in Madison in 1964, led to my emigration and recently to my becoming the first recipient of the new Byron Bird Award, given by the University of Wisconsin-Madison College of Engineering 'in recognition of an outstanding, widely accepted engineering research publication which has had a profound influence in the academic or industrial community.' "

1. Lodge A S. On the use of convected coordinate systems in the mechanics of continuous media. *Proc. Camb. Phil. Soc.* 47:575-84, 1951.
2. Oldroyd J G. On the formulation of rheological equations of state. *Proc. Roy. Soc. A* 200:523-41, 1950.
3. Huilgol R R. *Continuum mechanics of viscoelastic liquids*. New York: Halsted Press, 1975. p. 102-4.
4. Lodge A S. *Body tensor fields in continuum mechanics, with applications to polymer rheology*. New York: Academic Press. 1974. 319 p.