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Zames G. On the input-output stability of time-varying nonlinear feedback systems. Parts I and II. *IEEE Trans. Automat. Contr.* **AC-ll**:228-38**;** 465-76, 1966. [Dept. Electrical Engineering, Massachusetts Institute of Technology, Cambridge, MA]

This paper summarizes a feedback stability theory developed by the author during 1959-64. It introduces the concept of an extended space, and establishes three equivalent principles for input-output stability (or incremental stability): the 'small-gain,' 'positive-operator,' and 'conic-sector' theorems. Applications include the circle criterion, and a new proof of Popov's criterion. [The *SCI*® indicates that Part I of this paper has been cited over 145 times and Part II over 115 times since 1966.]

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"In the late 1950s, Norbert Wiener started a research project at MIT with the object of cracking that hard nut, Nonlinear System Theory. The group at the Research Laboratory for Electronics with which Wiener became associated included, among many other distinguished names, Jerome B. Wiesner, Y.W. Lee, Amar G. Bose, Don A. George, Irwin Jacobs, Martin Shetzen, and Harry Van Trees. Many subsequent developments in nonlinear systems involving Volterra series, Cameron-Martin expansions, Fox bases, etc., originated in that period.

"Shortly after joining the group, I took a functional-analysis course from I.W. Singer, and decided to apply the newly learned material to the problem of nonlinear feedback, for no better reason than that this seemed to be challenging virgin territory.

"The first breakthrough came in 1959 in 'Conservation of bandwidth in nonlinear operations,'[1] where the problem of recovering bandlimited signals that had been nonlinearly filtered was solved by using a contraction-mapping-based feedback scheme.

"By 1964 I had the makings of a substantial theory,[2-4] including the extended-space concept, circle theorem, positive operator theorem, and the notion of physical readability (now known as strict causality). The present paper was an attempt to unify this theory. Many of the ideas in the paper, notably the 'small-gain' approach, can be found in the widely circulated 1960 report,[2] essentially a part of my doctoral thesis.

"The paper was submitted in 1964, but took almost two years to appear in print. After losing one draft, referees found the paper too long, too short, simplistic, complicated, complained about style, demanded references, and almost rejected it. Nevertheless, the paper immediately drew much attention and, two years later, I won an American Automatic Control Council best paper award, nominally for another paper, but undoubtably in recognition of this work. In a competitive area, real results can be the hardest ones to publish!

"The paper is cited mainly for having introduced a then new approach to feedback systems, which has since been widely adopted, and which has led, more or less directly, to the current theories of 'large-scale weakly-coupled' systems, and 'diagonally-dominant' frequency-response methods."

1. **Zames G.** *Conservation of bandwidth in nonlinear operations.* Cambridge, MA: Massachusetts Institute of Technology, Research Laboratory for Electronics, October 1959, Quarterly PR55, p. 98-109.
2. ..... *Nonlinear operators for system analysis.* Cambridge, MA: Massachusetts Institute of Technology, Research Laboratory for Electronics, September 1960, Technical Report 370.
3. ..... Functional analysis applied to nonlinear feedback systems. *IEEE Trans. Circuit Theory* **CT-10**:392-404, 1963.
4. ..... On the stability of nonlinear, time-varying feedback systems. *Proc. NEC* **20**:725-30, 1964.