

# ***This Week's Citation Classic***

Gear C W. The automatic integration of ordinary differential equations. *Commun. ACM* 14:176-9, 1971.  
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**This paper presents the first automatic computer code for the solution of stiff or non-stiff differential equations. The techniques presented allow the code to select its own order and stepsize automatically with the goal of minimizing integration time. [The SCI® indicates that this paper has been cited over 130 times since 1971.]**

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"In the middle 1960s the problem of stiff differential equations was not well known to numerical analysts and there were no very general techniques available for their solution. Most practising scientists were forced to use analog computers to solve such problems, and there were serious proposals to build multi-million dollar analog computers for specific problems in chemical kinetics. At the time I was spending a summer at the Argonne National Laboratory studying digital computer methods for differential equations, and one

of the ana-log computer staff issued a challenge saying he had a problem that 'you digital computer types' would never be able to solve. This was far better incentive than anything else, and the peaceful research climate at Argonne allowed time to ponder the problem and realize that the backward differentiation method, a method previously discarded by numerical analysts, had characteristics that made it very suitable for this problem. In fact, this was a re-discovery of a method already published by some chemists 14 years earlier. The method was not in use because the difficulties of computer implementation had not been investigated.

"However, publication of the method of computer implementation in 1967<sup>2</sup> and 1968<sup>3</sup> did not cause any interest outside of the numerical analysis community, and it was not until the late George Forsythe of Stanford encouraged me to include a computer code in a book<sup>4</sup> I was writing on the subject that the method became widely known. The paper contains the same code as appears in the book, and between them brought the code to the attention of many people who found that they could now solve many problems with relatively little effort. Since then there have been many improvements in the ideas.<sup>5</sup> Principally, I think that the frequent citation of the paper is due to the vision of Forsythe who believed that the job of numerical analysts was to help scientists solve real problems, and that the only way that was going to be accomplished was by incorporating new ideas in easily-usable programs."

1. Curtiss C F & Hirschfelder J O. Integration of stiff equations. *Proc. Nat. Acad. Sci. US* 38:235-43, 1952.
2. Gear C W. Numerical integration of stiff ordinary differential equations. Report no. 221. Urbana, IL: University of Illinois Computer Science Department, 1967. 20 p.
3. ...., The automatic integration of stiff ordinary differential equations. (Morrell A J H, ed.) Information processing 68. Amsterdam: North Holland, 1969. p. 187-93.
4. ...., Numerical initial value problems in ordinary differential equations. Englewood Cliffs, NJ: Prentice Hall, 1971. 253 p.
5. Shampine L F & Gear C W. A user's view of solving stiff ordinary differential equations. *Soc. Ind. Appl. Math. Rev.* 21:1-17, 1979.