

# This Week's Citation Classic

**Bonner J C & Fisher M E.** Linear magnetic chains with anisotropic coupling. *Phys. Rev. A* 135:640-58, 1964.  
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**A novel numerical method is used to 'solve' the analytically intractable problem of the Heisenberg magnetic chain. The method estimates the properties of an infinitely large one-dimensional system by extrapolation of a sequence of exact calculations on finite chains of increasing length. [The SC<sup>®</sup> indicates that this paper has been cited over 355 times since 1964.]**

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"Work on the project featured in this work began in the summer of 1960. Michael E. Fisher, then a junior lecturer at King's College, University of London (and now Horace L. White professor of chemistry, physics, and mathematics, Cornell University), had just visited the US. There he discussed with Charles Kittel the germ of a novel theoretical method. Kittel thought the idea wouldn't work but Fisher believed otherwise. I was Fisher's first and unofficial graduate student. According to the rules of the University of London, only faculty of sufficiently senior status could officially supervise a graduate student. My official advisor was, therefore, Cyril Domb, subsequently Wheatstone professor of physics, King's College.

"Research in progress at King's College at that time consisted of pioneering attacks on problems in cooperative phenomena and statistical mechanics. A major aspect of this activity comprised extensive studies to determine the nature of the critical singularities characterizing a system undergoing a phase transition. This was done numerically by application of extrapolation techniques to high-temperature series expansions. Such techniques were yielding a wealth of information on Ising model behavior, but were much less successful for the physically more realistic Heisenberg model. The Heisenberg

model at that time was not exactly solvable in any dimension (except for limited results for the ground state energy in one dimension). It remains unsolved in two and three dimensions. The project, therefore, was to obtain, for the first time, reliable information on the isotropic and anisotropic Heisenberg model, albeit in one dimension. The method was basically numerical. The properties of small chains (rings) of spins with  $N = 2$  through 11 were calculated exactly, and then a variety of extrapolation techniques were devised to infer the behavior in the thermodynamic limit of a wide variety of thermodynamic properties of the magnetic system. The computations were done on one of the first successful scientific digital computers, the University of London's Ferranti 'Mercury,' mostly between 1960-2. The calculations involved the diagonalization of  $2^N \times 2^N$  Hamiltonian matrices, after substantial reduction in size by use of symmetry properties of the Hamiltonian.

"The paper was initially received with either scepticism or curiosity, since it made no contact with accepted analytical approximate methods. The validity of the numerical, extrapolated results was called into question, not least by the referee who referred to (in paraphrase) 'a poorly-supported bundle of conjectures with no basis in reality!' Others tended to view a one-dimensional model calculation as merely a toy, devoid of practical application.

"Since then, the accuracy of the results, as determined both by subsequent exact calculations on the general anisotropic Heisenberg chain and by close agreement of theory and experiments performed on quasi-one-dimensional magnetic insulators, has been established. The success of this work, therefore, has lain the provision of a wide range of reliable numerical answers to a problem which was analytically intractable. This provided a big stimulus to experimental physicists and more recently to coordination chemists and to theorists by serving as a check not just on approximate analytic methods but on subsequent exact analytic work."