

This Week's Citation Classic

Whitham G B. A general approach to linear and nonlinear dispersive waves using a Lagrangian. *J. Fluid Mech.* **22**:273-83, 1965.

A general theory is developed to describe the propagation of modulations on a nonuniform wavetrain. The key to the generality and to the smooth manipulation of analytic details is the use of a variational formulation and the corresponding Lagrangian. [The SC[®] indicates that this paper has been cited over 150 times since 1965.]

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"In teaching the concept of group velocity and emphasizing its fundamental importance, I was struck repeatedly by the strong limitations to linear wave systems. Yet, many of the interesting wave problems, in fluid mechanics and water waves for example, are governed by nonlinear equations in the original formulation. I also knew from previous experience with shock wave problems that important aspects can be completely lost in going to the linear approximation. This work started with a search for a nonlinear definition of group velocity and some approach to its role in the nonlinear case.

"The first success was to use conservation equations to obtain modulation equations for the propagation of changes in overall quantities in amplitude, wave number, frequency, etc., and suppress the detailed oscillations. These equations were sometimes hyperbolic and their characteristic velocities were a natural generalization of the group velocity. However, the averaging procedure was taken over oscillatory solutions involving at

best elliptic functions and their generalizations, to replace the usual sines and cosines. The manipulations became too complicated to get explicit results until I noticed that all the quantities of interest in the modulation equations could be obtained from some kind of master function and its derivatives. Eventually, and this took some months, I realized that the master function was equivalent to a Lagrangian, and that the whole derivation could be presented with remarkable simplicity in terms of the corresponding variational principle. So much so that this development even gave new general ways to derive results in the supposedly completely well known linear case.

"A major obstacle was a stubborn belief that the modulation equations should be hyperbolic corresponding to real group velocities. Yet in some cases the linear group velocity split into two complex conjugates. I kept looking for the mistakes. Finding none, my next view was that this elliptic case only arose in unstable situations of little practical interest. Finally, it became clear that the instability interpretation was correct and actually occurred in important problems. Other developments opened up from the variational approach automatically. These included the further discussion of general questions for wave systems in terms of the Lagrangian without specifying the particular system; simple derivation of conservation equations; adiabatic invariants for waves, analogous to those in dynamical systems; immediate extension to nonuniform media with no change in the formalism; the mathematical device of applying perturbations expansions directly in the variational principle rather than its Euler equations.

"Presumably it is the variety of these new aspects that has led to many citations of the paper.

"The work was carried out at the California Institute of Technology where the stimulation of environment and faculty is much appreciated."