CC/NUMBER 31
JULY 30, 1979

This Week's Citation Classic

Shepard R N. The analysis of proximities: multidimensional scaling with an unknown distance function, I & II. *Psychometrika* **27**:125-40, 219-46, 1962. [Bell Telephone Laboratories, Murray Hill, NJ]

A computer-based method of 'nonmetric' multidimensional scaling is described (Part I) and applied to empirical data (Part II). Knowledge, of merely the rank order of the n(n-1)/2 measures of similarity or 'proximity' between n objects, interpreted as n points in a coordinate space, permits a generally unique determination of the dimensionality of the space and the metric configuration of the points. [The *Science Citation Index*® (*SCI*®) and the *Social Sciences Citation Index*™ (*SSCI*™) indicate that these papers have been cited over 445 times since 1962.]

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May 31, 1979

"I have always regarded the development reported in this paper as one of my most original and significant accomplishments. It was largely responsible, I'm sure, for my later election to the presidency of the Psychometric Society and it, along with just two other of my contributions, was singled out for specific mention in the citation of the Distinguished Scientific Contribution Award that the American Psychological Association bestowed on me in 1976.

"The other two contributions referred to in that award were my computer generation of a paradoxical sequence of endlessly rising tones,[1] and my recent demonstrations of the analog character of 'mental rotation.'[2] I mention these two other developments because all three shared two notable circumstances: (a) In each case the basic idea came (typically just upon awakening) in a flash of geometrical intuition that carried great conviction, even though verification had to await a major effort of computer programming and experimental investigation; (b) In each case the empirical results, when finally obtained, surpassed even my own highest expectations. In the case of nonmetric multidimensional scaling, it was the precision with which metric structure could be recovered from purely ordinal proximity data that exceeded my fondest dreams.

"The idea of representing objects (such as colors, sounds, shapes, faces, word meanings, etc.) as points in space, in such a way that the distances between the points represented the perceived similarities between the objects, had occurred to me while I was an undergraduate student in 1951. But it was not until ten years later, when I gained access to the powerful computing facilities at the Bell Telephone Laboratories, that I conceived of an iterative process for reconstructing the implied spatial configuration even when the form of the monotone function relating similarity and distance was completely unknown.

"After a period of trial-and-error adjustment of the parameters of the iterative process, success came with dramatic suddenness on March 17, 1961. According to the computer log, it was at precisely 2.33 p.m. EST on that day that the iterative process first converged to a stationary configuration, revealing a remarkably exact recovery of an underlying test configuration. The excitement of that moment was rivaled only by the birth of my daughter on the very next day. Since then my daughter has developed into a fine young woman; and, thanks in part to the subsequent contributions of my mathematical colleague Joseph Kruskal, nonmetric multidimensional scaling is now finding wide application throughout the cognitive, behavioral, and biomedical sciences."[3]

1. **Shepard R N.** Circularity in judgments of relative pitch. *J. Acoust. Soc. Amer.* **36**:2346-53, 1964.
2. **Shepard R N & Metzler J.** Mental rotation of 3-dimensional objects. *Science* **171**:701-3, 1971.
3. **Kruskal J B.** Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis. *Psychometrika* **29**:l-27. 1964.