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This Week's Citation Classic

Peaceman D W & Rachford H H, Jr. The numerical solution of parabolic and elliptic differential equations. *J. Soc. Ind. Appl. Math.* **3**:28-41, 1955.

This paper introduces an alternating-direction implicit procedure for solving the heat flow equation in two space dimensions. Analysis shows it to be stable for any size time step and computationally more efficient than other implicit methods. Also, the method is applicable to the iterative solution of difference equations for second-order two dimensional steady-state problems and requires significantly fewer arithmetic operations to solve Laplace's equation than other iterative methods. [The *SCI*® indicates that this paper was cited 216 times in the period 1961-1977.]

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"Thank you for your invitation to be Citation Classics contributors. Our collaboration began as M.I.T. chemical engineering graduate students .After graduation we went to work for Humble Oil's Production Research Division, whose management wanted to develop computer models of oil and gas reservoir processes. We became fascinated by the effectiveness of implicit (backward) difference methods for solving flow problems in one space dimension (1-D). But 1-D rarely provides enough realism for reservoir simulation, so the incentive was strong to pursue 2-D and 3-D solutions. This goal was made doubly elusive by our computing resources: an electromechanical kludge called the CPC (Card Programmed Calculator), from which could be coaxed, at most, five floating-point operations per second. Its 56-word data memory was appropriately dubbed the "ice box "

"At the time, we were considering a flow problem in cylindrical (r-z) coordinates and deploring the inefficiency of solving 2-D implicit difference equations This, and the inherent r-z asymetry, suggested another approach: keep the implicit flavor but improve the efficiency by making the difference equation backward in only one direction and forward in the other. Then, to compensate the slighted direction, the next step we would reverse the backward/forward directions; thus was born the name *Alternating Direction* (AD).The idea arose casually late on December 30, 1953, but the next morning our thoughts were not on the calendar. Overnight we had independently analyzed the new procedure and found it promising. Our colleagues' initial pronouncements were distinctly negative, but the analysis was there; the method works.

"Long before, one of us and his wife had planned a New Year's Eve party for that evening. Needless to say, we talked much shop that night, much to the distress of our lovely hostess. Some years later she finally forgave us when, browsing through a library, her husband pulled out a textbook "at random" and showed her a section on the Peaceman-Rachford A-D procedure.

"The method was first tested using the CPC on the heat flow equation in a square with a 14 by 14 grid. Most of the data memory was on cards punched out each half-step, with the direction being alternated by using a card sorter. It was a long time before megabyte memory!

"The idea had many offshoots. Douglas showed how it related to the Crank-Nicolson difference equation. It was soon extended to 3-D. The Russian school espoused the related fractional step or splitting method. In its time, it was much used in oil reservoir and nuclear reactor simulation. And during the recent revival of Galerkin methods, Douglas and Dupont devised A-D Galerkin methods for n-D hyperbolic and parabolic problems."