

This Week's Citation Classic

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Harter H L. Critical values for Duncan's new multiple range test.

Biometrics 16:671-85, 1960.

[Aeronautical Research Laboratories, Wright-Patterson Air Force Base, OH]

The author tabulates critical values for Duncan's new multiple range test, at significance levels α [protection levels $P = (1 - \alpha)^p$], of p successive values out of n ordered means, for $\alpha = 0.10, 0.05, 0.01, 0.005, 0.001$; $p = 2(1) n$; $n = 2(1) 20(2) 40(10) 100$ and degrees of freedom $v = 1(20), 24, 30, 40, 60, 120$." [The SC[®] indicates that this paper has been cited over 150 times since 1961.]

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"In 1954 I began a study of error rates and sample sizes for multiple comparison tests, especially those based on the studentized range. The results of this study were published in a 1957 journal article.¹ During the course of this work, I discovered sizeable errors in the published tables of critical values for Duncan's new multiple range test, and smaller ones in those of percentage points of the studentized range used as critical values for Tukey's studentized range test, the Newman-Keuls test, and Tukey's compromise X procedure.

"In order to correct and extend these tables it was necessary to compute new tables of the probability integral of the studentized range, more extensive and more accurate than those of Pearson and Hartley (1943).² This in turn required the computation of more extensive and more accurate tables of the probability integral of the range than those of Pearson and Hartley (1942).³ The new tables of percentage points of the studentized range were published in another 1960 journal article.⁴ The complete results, including the tables of probability integrals, were published in two 1959 technical reports. These results, together with corrected and extended tables of error rates, were reproduced in a 1970 book.⁵

"This work was performed at the Aeronautical (later Aerospace) Research Laboratories (ARL), with the active encouragement of Paul R. Rider, then chief statistician at ARL. The programming for the Univac Scientific (ERA 1103A) computer was done by Donald S. Clemm and Eugene H. Guthrie, based on numerical analysis by Gertrude Blanch. These and other colleagues deserve much credit for successful completion of the mammoth undertaking.

"I attribute the frequent citation of this paper to the fact that it provided the first accurate critical values for the widely used new multiple range test proposed by David B. Duncan in an even more frequently cited 1955 paper featured earlier in their series."⁶

1. **Harter H L.** Error rates and sample sizes for range tests in multiple comparisons. *Biometrics* 13:511-36, 1957.
2. **Pearson E S & Hartley H O.** Tables of the probability integral of the 'studentised' range. *Biometrika* 33:89-99, 1943.
3. **Pearson E S & Hartley H O.** The probability integral of the range in samples of ζ observations from a normal population. *Biometrika* 32:301-10, 1942.
4. **Harter H L.** Tables of range and studentized range. *Ann. Math. Statist.* 31:1122-47, 1960.
5. **Harter H L.** *Order statistics and their use in testing and estimation. Volume 1: Tests based on range and studentized range.* Washington, DC: US Government Printing Office. 1970. Out of print, but paper-bound and microfiche copies available from NTIS. (AD-A058262).
6. **Duncan D B.** Multiple range and multiple F tests. *Biometrics* 11:1-42, 1955.