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Citation Classics

Duncan D B. Multiple range and multiple *F* tests. *Biometrics* 11:1-42, 1955.

This new multiple range test, for determining the homogeneity of a set of n values in an analysis of variance in a population, combines the proposals by Newman in 1939 and Keuls in 1952 with the author's earlier multiple comparison tests. A series of tests paralleling the methods of multiple range tests have been termed multiple F tests, which use "protection levels based on degrees of freedom." An F test alone, it is demonstrated, "falls short of satisfying all of the practical requirements." One of several test procedures examined is termed the least-significant-difference (or L.S.D.) test. [The *SCI*® indicates that this paper was cited 3,610 times in the period 1961-1975.]

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"I am naturally glad and encouraged to have a 'most cited' paper and to learn of the frequent use of the DMR (my multiple range) rule.

"The 1955 DMR rule was a modified version of my earlier 1951 DMF (multiple F) rule. Both of these rules ranked in conservatism and power between the less conservative 1935 FLSD (Fisher LSD) rule and the more conservative 1939 NMR (Newman MR) rule. By using F tests, the DMF rule could be used to test comparisons (subsequently called contrasts by Scheffe) as well as pairwise differences. However, the multiple use of F tests was more cumbersome than that of range tests, and the DMF rule received much less attention.

"In between these rules came the TLSD (Tukey LSD) rule of 1952 and the SLSD (Scheffe LSD) of 1953 Both of these were based on the use of experimentwise levels making them much more and very much more conservative and less powerful than even the NMR rule.

"The frequent use of the DMR rule has been encouraging in the support it has given for my less conservative approach.

"In 1955 when I published the DMR rule I had not been able to finish a multiple decision theory approach to the problem which I had started in my thesis in 1947.

"In this kind of approach, which can be held to be the ideal, it seemed reasonable from the start to choose an additive loss function. That is, a loss function whereby, roughly speaking, the seriousness of the error made by any joint decision is scored in proportion to the number of individual differences about which it is wrong. In presenting the DMF and DMR rule I had been influenced by this approach but had also had to resort to arguments of a more ad hoc nature.

"On taking up the decision theory approach again in 1962, I was able to show that, by using a super normal population prior model $\mu_i \sim N(\mu_o, ó_\mu^2)$ for the true treatment means, the optimal rule was a 'k-ratio' LSD rule with a LSD which depends on the variance ration $ró_\mu^2/ó_e^2$, $ó_e^2$ being the error variance. Subsequently in 1965 it was exciting to find that, by putting conjugate X^2 priors on $ó_\mu^2$ and $ó^2$ the LSD for this rule could be made to depend directly on the observed F ratio.

"Joined by R.A. Waller in 1969, we were able, by switching the prior on $ó_\mu^2$ to being an independent conjugate X^2 on $ó_T^2 = ró_\mu^2 + ó_e^2$, to derive and table the precise t values for the k-ratio LSD Bayes rule for all small sample values of the numerator and de-nominator degrees of freedom of the ob-served F ratio.

"By having its t value depend on the F ratio the k-ratio LSD rule is adaptive. It can vary, in an intuitively pleasing way, all of the distance from being conservative like an experiment-wise rule when the F is small to being less conservative and more powerful than even a comparisonwise rule when F is large. This rule, which I showed later in 1975 can be used on all contrasts as well as pairwise differences, [1] is the one I now recommend in place of all of the earlier rules including the cited DMR rule ".

1. **Duncan D B.** T tests and intervals for comparisons suggested by the data. *Biometrics* 31:339-59, 1975.